Finding Domain for Functions

-This tutorial will focus on finding the Domain algebraically for Polynomial, Rational, and Radical Functions. (Knowledge of Interval Notation is assumed.)

What is Domain and why does it matter?

-The Domain for a function is its input, just as its output is called Range.

-This matters because in math you can't put a number into a function that will violate the rules of logic, when you have a situation like this, you remove all numbers that violate the rules from the Domain. This is called a domain restriction.

-In the Real World the need for this type of thinking is most apparent in the world of computer programming.

<u>**Polynomial Functions**</u>

-Polynomial functions are power functions that range from linear up, below are 3 examples of different polynomial functions.

$$f(x) = 2x - 1 \ , \qquad f(x) = 2x^2 - x + 4 \ , \qquad f(x) = 2x^4 - x^3 + 2x - 8$$

-In terms of domain, polynomial functions are notoriously friendly and unless otherwise imposed, have no domain restrictions.

-This means that there is no real number that cannot be plugged into a polynomial function, in other words, the domain is all real numbers, all the time.

-Using interval notation, the Domain for polynomial function is $(-\infty, \ \infty)$.

Example 1: Find the domain for $f(x) = -3x + 7$.

SOLUTION: Since this is a polynomial function, the domain is $(-\infty, \ \infty)$.

Example 2: Find the domain for $f(x) = \frac{2}{3}x^2 - 2.5x + 3$.

SOLUTION: Since this is a polynomial function, the domain is $(-\infty, \ \infty)$.

Example 3: Find the domain for $f(x) = 2x^5 - 3x^4 + x^3 - 5x^2 + 2x - 1$.

SOLUTION: Since this is a polynomial function, the domain is $(-\infty, \ \infty)$.

Example 4: Find the domain for $f(x) = -5$.

SOLUTION: Since this is a linear polynomial, the domain is $(-\infty, \ \infty)$.

Rational Functions

-A rational function is a function that contains variables in the denominator as well as the numerator. Below are a few examples of rational functions.

$$f(x) = \frac{2}{x-1}, \qquad f(x) = \frac{2x-3}{3x-2}, \qquad f(x) = \frac{2x^2-3x+1}{x}$$

-It may be tempting to think that every function that contains fractions is rational, but this is not always the case. For example, $f(x) = \frac{2x-3}{4}$, looks rational, but upon closer inspection, we see that there is no variable in the denominator, so this is a polynomial.

The Important Rules for Rational Functions

-The only restriction on the domains of rational functions is that there can never, I repeat, never, be a 0 in the denominator.

-Dividing by zero would cause our function to misbehave in a terrible way.

-Any number that would create a 0 in the denominator, **MUST** be removed.

-To find the number that will create 0 in the denominator, we set the denominator equal to 0 and solve.

-Whatever number is our solution, must be removed from the domain.

Example 1: Find the domain for $f(x) = \frac{1}{x-3}$.

SOLUTION: To find the value that would create 0 in the denominator, we set the denominator equal to 0 and solve. So, $x - 3 = 0$, and to solve this we simply add 3 to both sides getting $x = 3$. This means that 3 MUST be removed from the domain.

When we remove 3 from the number line, we get a domain of, $(-\infty, 3) \cup (3, \infty)$.

Example 2: Find the domain for $f(x) = \frac{2x-4}{4x-3}$.

SOLUTION: Even though we have terms in the numerator, none of them will restrict the domain, so we once again set the denominator equal to 0 and solve.

$$4x - 3 = 0 \qquad \rightarrow \qquad 4x = 3 \qquad \rightarrow \qquad x = \frac{3}{4}$$

This means that $x = \frac{3}{4}$ MUST be removed from the domain of the function.

Therefore, the domain is $\left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)$.

Example 3: Find the domain for $f(x) = \frac{x-1}{x^2+x-6}$.

SOLUTION: Just like in previous problems, we must set the denominator equal to 0 and solve, however, we are going to have to remove two numbers from the domain this time.

First, we set $x^2 + x - 6 = 0$. This is a quadratic equation so to solve it we can factor, or if you prefer, you may use the quadratic formula. If we factor, we get $(x-2)(x+3) = 0$. This will give us solutions of $\{2, -3\}$. These numbers must be removed from the domain.

Once we remove -3 and 2, we have a domain of $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

Radical Functions

-Radical Function are a large family, in this tutorial, we will focus on the two types of radical functions most often seen by students, they are the square root functions, and cube root functions.

Square Root Functions

- As their name implies, square root functions are functions that contain a square root, some examples of square root function are listed below.

  $$f(x) = \sqrt{x+3} \qquad f(x) = \sqrt{2x-3}$$

- With square root functions, the most important thing to keep in mind is that we CANNOT have a negative in the radicand, therefore, every number within the radicand must be greater than, or, equal to, 0. We use this simple inequality to find the domain.

Example 1: Find the domain for $f(x) = \sqrt{2x-7}$.

SOLUTION: Since we know that whatever is within the radicand is greater than or equal to 0, we set it up as an inequality, $2x - 7 \geq 0$. Next, we add 7 to both side getting $2x \geq 7$, now all we must do is divide both sides by 2 and have $x \geq \frac{7}{2}$. This means that every x must be greater than or equal to $\frac{7}{2}$. Therefore, in interval notation, our domain is $\left[\frac{7}{2}, \infty\right)$.

Example 2: Find the domain for $f(x) = \sqrt{-x-3}$.

SOLUTION: We set up our inequality as $-x - 3 \geq 0$. Next, we add 3 to both sides getting $-x \geq 3$. Now, we divide by -1, remember that that will change the direction of the inequality, $x \leq -3$. Therefore, $(-\infty, -3]$ is the domain.

Example 3: Find the domain for $f(x) = \frac{3}{\sqrt{2x-3}}$.

SOLUTION: This is an interesting problem because it combines both radical and rational functions. This truth is that it really isn't much different from the other problems as we will see below. The difference here is that we will set up out inequality like $2x - 3 > 0$, without the equal to, because we CANNOT have 0 in the denominator. We will solve the inequality by first adding 3 to both sides, $2x > 3$, now we will divide both sides by 2 to get $x > \frac{3}{2}$. Therefore, the domain for this function is $\left(\frac{3}{2}, \infty\right)$.

Cube Root Functions

- Cube root functions are functions that contain a cube root, below are some examples
$$f(x) = \sqrt[3]{x+3} \qquad f(x) = \sqrt[3]{2x+4}$$
- While cube root functions look very similar to square root functions, they actually behave very differently. You may remember when learning about cube roots that you **<u>can</u>** have a negative inside a cube root. Because of this simple fact the domain for a cube root function will in most cases be $(-\infty, \infty)$.

Example 1: Find the domain for $f(x) = \sqrt[3]{3x-2}$.

SOLUTION: Since this is just a simple cube root function with nothing unusual going on, the domain is simply $(-\infty, \infty)$.

Example 2: Find the domain for $f(x) = \sqrt[3]{5 - \frac{1}{2}x}$.

SOLUTION: This is just like the other problem, this is a simple cube root with nothing unusual going on, so the domain is simply $(-\infty, \infty)$.

Example 3: Find the domain for $f(x) = \frac{1}{\sqrt[3]{3x-7}}$.

SOLUTION: This is an atypical problem because the cube root is in the denominator, therefore we know that we cannot have 0 in denominator so we will set what is inside the cube root equal to 0 and then solve. Therefore, $3x - 7 = 0$, now we add 7 to both sides getting, $3x = 7$, and lastly, we divide both sides by 3 getting $x = \frac{7}{3}$. Since we know that we need to remove $\frac{7}{3}$ from the domain, the domain of our function is $\left(-\infty, \frac{7}{3}\right) \cup \left(\frac{7}{3}, \infty\right)$.